Joint Optics/Signal Processing Design for Computational Diffractive Sensing and Imaging

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Abstract: Computational optical imaging has relied almost entirely upon the integration of computation with reflective and refractive optics. Computational *diffractive* optics relies upon diffractive optical elements and requires new principles and techniques for joint design.

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1. Background and history

Computational optical sensing and imaging has relied on the deep integration of optics (traditional reflective and refractive elements, apertures, shutters, structured light, etc.) with signal processing in order to yield digital images or estimates of visual information such as scene motion or object location. The more the digital imaging datapath includes signal processing, the greater is the design freedom in the optics. Recently, this general approach has been extended to the design of systems containing solely *diffractive* optical elements, leading to ultra-miniature diffractive optical imagers and sensors. [1–6] Methods of joint optics/signal processing co-design applicable to systems with refractive and reflective elements must be modified before they can be applied to systems with purely diffractive optical elements.

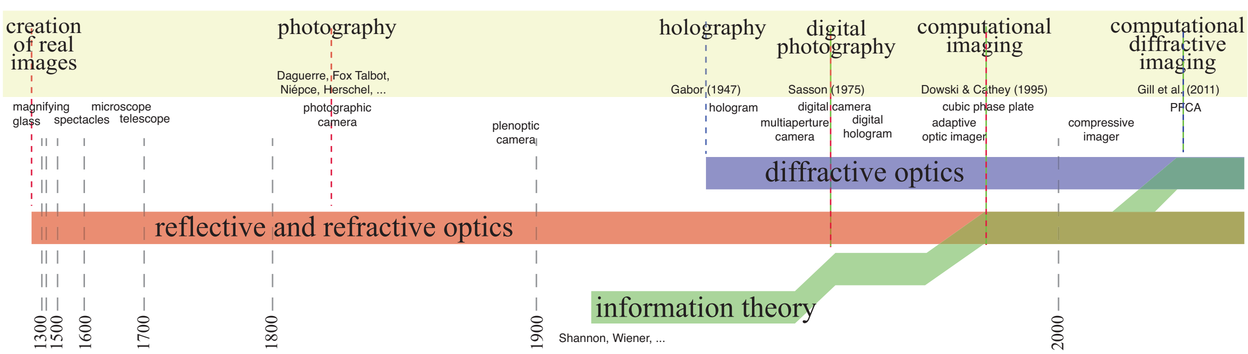

Fig. 1. Timeline of computational optical imaging showing several conceptual milestones. Images were created by pinhole cameras and camera obscurae for centuries, but in the Medieval era focusing lenses and mirrors were understood sufficiently that magnifying glasses, microscopes and telescopes could be built. In the early 19th century real images were fixed chemically, through mercury and ultimately silver-halide—the birth of photography. A separate historical thread includes signal theory, information theory, and innumerable developments in both algorithms and hardware in computer science which was integrated with optics in 1975 to yield the digital still camera and its many descendants. [7] The era of true computational imaging—in which optics and image processing were designed for end-to-end performance—can be dated to Dowski and Cathey's work on cubic phase plates. [8, 9] The earliest imaging relying fundamentally upon diffraction was holography, in 1947. [10] Although computer-generated holograms became common in the 1980s, until very recently, computing has not been included into true holographic image acquisition from a physical object. Computation and diffractive optics were first deeply integrated in the Planar Fourier Capture Array (PFCA) of Alyosha Molnar and his team, [11] which was the inspiration behind our work.

2. Joint design in computational diffractive sensing and imaging

In traditional sequential design for computational optical sensing, one designs the optics to achieve some optimum of an optical merit function, and one then designs the signal or image processing for optimal end-to-end performance. Because the optical/digital design space is large and the merit function is complicated, this sequential method often leads to sub-optimal overall imaging performance (Fig. 2). Robinson and Stork showed how true joint design—where both the optical and image processing parameters of the overall system were computed *simultaneously*—could lead to end-to-end performance superior to that achievable through the sequential design method, or could lead to equivalent imaging performance with simpler optics. [12, 13] A computationally tractable merit function for joint design in the case of linear optics and linear signal processing is the predicted mean-squared image error, that is,

$$MSE = \frac{1}{n}Tr[(\mathbf{R}(\Theta_{IP})\mathbf{A}(\Theta_{opt}) - \mathbf{I})\mathbf{C}_s(\mathbf{R}(\Theta_{IP})\mathbf{A}(\Theta_{opt}) - \mathbf{I})^t + \mathbf{R}(\Theta_{IP})\mathbf{C}_n\mathbf{R}^t(\Theta_{IP})], \tag{1}$$

where $\mathbf{R}(\Theta_{IP})$ is the image processing filter (weights on its taps), $\mathbf{A}(\Theta_{opt})$ is the optical transfer matrix, $\mathbf{C}_s$ is the source covariance, $\mathbf{C}_n$ is the pixel-wise noise covariance, n is the number of sensor pixels and ^{t} denotes matrix transpose.

Figure 2 shows schematically how sequential search over the optical parameters, Θ_{opt}, then over the image processing parameters, Θ_{IP}, can lead to suboptimal value of the overall digital merit function, $C(\Theta_{opt}, \Theta_{IP})$, where here low values are preferred. True joint design, in which both optical and image processing parameter sets are adjusted simultaneous by gradient descent of Eq. 1 can lead to superior performance—lower $C(\Theta_{opt}, \Theta_{IP})$. [12] Of course, there may be local minima requiring that multiple searches be employed, each with a different initial condition. Here design heuristics for initializing design searches for electro-optical systems will have to be developed.

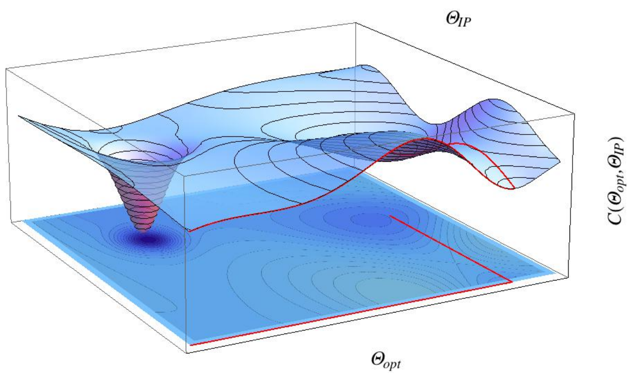

Fig. 2. A schematic of an end-to-end imaging merit or cost function $C(\Theta_{opt}, \Theta_{IP})$ as a function of the optical and image processing parameters, Θ_{opt} and Θ_{IP}. In traditional sequential design, the optics is first designed to yield a best performance or minimum cost, then the optical system is held fixed while the image processing parameters are designed to yield the minimum cost—as indicated by the red trajectories. Because the function is complex and non-separable, sequential design may not find the overall global optimum of the merit function, whereas true joint design may. Joint design produces superior computational imagers to sequential design, at least for some simple imaging systems. [12]

Equation 1 and related performance measures can be applied to classical ray optics of reflective and refractive optics with linear digital signal processing because the merit function depends continuously with optical parameters such as surface curvatures, lens separations, indices of refraction, and so on. For a number of reasons, the application of such methods to systems where light must be analyzed as a wave (diffractive optics) is far more difficult and problematic. First, the end-to-end merit function may be periodic with respect to certain design variables which control diffractive or interference phenomena, thus making the parameter search prone to numerous local minima. Second, the required simulation of wave phenomena in the inner loop of an optimization routine is computationally expensive for realistic two-dimensional gratings relevant to diffractive imaging. Given a candidate optical system matrix $\mathbf{A}(\Theta_{opt})$, the optimal image processing parameters can be computed as follows: If the output is $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{n}$, where $\mathbf{x}$ is the input scene and $\mathbf{n}$ noise, then the MSE-optimal estimate of the scene is $\hat{\mathbf{x}} = (\mathbf{A}^t\mathbf{A} + \gamma\mathbf{I})^{-1}\mathbf{A}^t\mathbf{y} \equiv \mathbf{R}(\Theta_{IP})\mathbf{y}$, where γ is a Tikhonov regularization parameter (Fig. 2). Third, constraints are difficult to impose, and computationally expensive to monitor during search. Fourth, while traditional joint design can exploit existing design tools such as *Zemax*—with their user-defined macros and operands—there are no analogous software tools for diffractive imaging optics. (Perhaps software tools for computational holography may be modified for such purposes.) Fifth, while optical design with traditional reflective and refractive elements can exploit heuristics (rules of thumb) for starting conditions, [14] there are few if any such heuristics for diffractive imaging systems. Consequently, full searches must be started from a broad range of incident system configurations. Finally, there is a difficulty that plagues both ray and wave imaging systems: the lack of an appropriate end-to-end merit function—one that expresses visually relevant criteria and is computationally tractable.

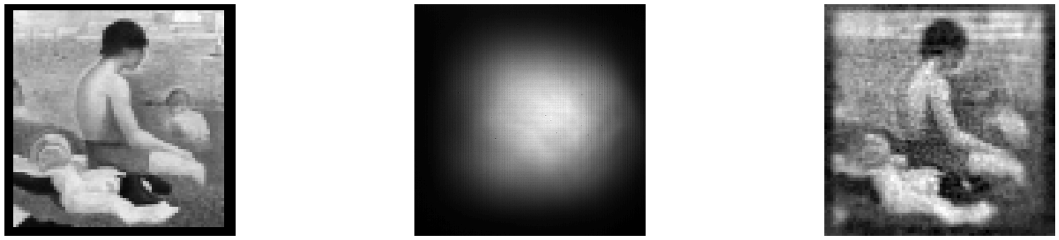

Fig. 3. Left: The input, a grayscale detail of Georges Seurat's *Baignade Asnière* (1884). Center: The raw 400 × 400 sensor signals. Right: the final 80 × 80-pixel digital image computed from sensor signals using Tikhonov regularization with an total variation-norm penalty.

True joint design for diffractive electro-optical imaging and sensing systems may be practical, at present, under highly constrained situations. For instance, one can lower the computational burden by altering a few spatial parameters in a one-dimensional phase anti-symmetric binary phase grating, with automatic update of image processing parameters, all for optimizing an image metric based on the end-to-end modulation transfer function. [3, 4] Here the requirement that the imager is invariant to manufacturing variations in overall grating thickness constrains the optical search space to spatial parameters that ensure the grating possesses binary phase anti-symmetry. [2]

3. Conclusions

Broad principles of computational imaging can be applied to sensors and imagers employing only *diffractive* optics but this approach is computationally feasible only under highly restrictive conditions. Future work will explore the relative merits of joint versus sequential design of diffractive computational sensors and imagers.

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